Intro to Logic

Worksheet

Week 13.

Topic: Bi-Conditionals; Review

Last week we learn that some conditional sentences “go in both directions” and some conditional sentences do NOT go in both directions.

For example, the conditional sentence below does NOT work in both directions:

If you live in NY, then you live in the US. N->U True.

If you live in the US, then you live in NY. U->N \* This is clearly not true.

But some conditional sentences go in both directions:

If this is an atom of oxygen, then this atom has eight protons. O -> E

If this atom has eight protons, then this atom is an atom of oxygen. E - > O

Let’s see if this sentence goes in both directions:

“If today is Monday, then tomorrow is Tuesday.” M -> T

Here the letter M will stand for “Today is Monday.”

The letter T will stand for “Tomorrow is Tuesday.”

Again, the symbolic expression M->T reads:

If today is Monday, then tomorrow is Tuesday.

But reversing the order of the antecedent and the consequent will still give us a true claim:

If tomorrow is Tuesday, then today is Monday.

For comparison, notice that the reversal does not work for:

If you have a dog, then you have a pet. D -> P (True)

If you have a pet, then you have a dog. P -> D (Not True)

So using here the double arrow will not work. D <-> P is not true.

Last week we also said that whenever the reversal works, we can use double head arrow, as in: M<-> T which means :If today is Monday, then tomorrow is Tuesday, and if tomorrow is Tuesday, then today is Monday.

In short, this sentence is a bi-conditional because Tuesday comes only after Monday and only Monday is before Tuesday.

Now your turn.

Let the letter W stand for “Yesterday was Wednesday.”

Let the letter T stand for “Today is Thursday.”

Let the letter F stand for “Tomorrow is Friday.”

Now, explain how you would read the symbolic notations below:

1. W <-> T
2. T <-> F
3. W <- > (T & F)

Now that we reviewed bi-conditionals, let’s practice deriving conclusions from various premises.

In this practice exercise, we will try to use different methods we have learned so far, including the BE (Bi-Conditional Equivalence).

Practice.

Example one:

1. M <- > T
2. – T
3. M v S

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What can we derive from these two premises? Let’s say the premises are about days of the week. So the first bi-conditional says: Only if Today is Monday, then tomorrow is Tuesday. And the second premise says that tomorrow is not Tuesday.

First let’s take apart premise 1, like this:

1. M <- > T
2. – T
3. M v S

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4. (M-> T) & (T->M) 1, BE (We derived this conjunction from line 1 by a

method called Bi-conditional Equivalence).

Now let’s use the method called simplification to reduce the conjunction. The conjunct we are interested in is the first one, the one in green: (M->T). Notice the entire parenthesis is one single conjunct. Notice the second parenthesis is a second conjunct. Two conjuncts give us a conjunction. So let’s write the step like this:

5. M -> T 4, Simp

Is that all? No. Remember what we are told in the second premise. The second premise (line 2) says: negative T. So now we can implement the MT method to derive –M, since line 5 says M->T, and line 2 says – T. So by MT we get – M, like this:

6. – M 2, 5 MT

Is that all? No. Look at premise 3. It says It’s either Monday or Sunday. Well since we now know that it’s not Monday, then according to the given premise in line 3, we must conclude by Disjunctive Syllogism that it’s Sunday, like this:

7. S 3, 6 DS

Your turn:

a)

1. A <- > B
2. – B
3. A v C

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4.

5.

6.

7.

b)

1. X <- > Y
2. – Y
3. X v Z

\_\_\_\_\_\_\_\_\_

4.

5.

6.

7.

More practice examples:

Let’s say the symbols below read as:

1. C <-> I Only Children are truly Innocent.

2. - C But John is not a child.

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To deduce the correct letter from the premises above, first we need to say what the bi-conditional in line one is equivalent to, like this:

3. (C -> I ) & (I -> C) 1, BE

Now we need to take apart the conjunction. We can already see that the second conjunct, the one in orange will be more useful, since we will be able to apply the MT method. So let’s write that we are simplifying the conjunction, and then that we can derive –I by MT, like this:

4. I -> C 3, Simp

5. –I 2, 4 MT

Now your turn. Create two premises, one of which is a bi-conditional. Be sure to supply your scenario (say what your letters stand for). What does your second premise say?

1.

2.

\_\_\_\_\_\_

More practice problems.

1. ( A v B) & C Premise 1
2. A-> Y Premise 2
3. - ( Y & C ) Premise 3

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What can we do with these three premises? We can start by doing something to last premise. We can use De Morgan’s Law and convert premise 3 into – Y v – C, like this:

1. ( A v B) & C Premise 1
2. A-> Y Premise 2
3. - ( Y & C ) Premise 3

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4. – Y v – C 3, DM

Next, remember that we can simplify conjunctions. What we have in line 1 is indeed a conjunction. The second conjunct is the letter C, which you can already notice will be useful for cancelling out the negative C is line 4. So let’s first write that we are simplifying line 1, like this:

1. ( A v B) & C Premise 1
2. A-> Y Premise 2
3. - ( Y & C ) Premise 3

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4. – Y v – C 3, DM

5. C 1, Simp

Now, let’s use DS to cancel out the C’s, like this:

1. ( A v B) & C Premise 1
2. A-> Y Premise 2
3. - ( Y & C ) Premise 3

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4. – Y v – C 3, DM

5. C 1, Simp

6. – Y 4, 5 DS

Now let’s look at premise 2. It says that IF we get A, then we get Y. But since we obtained negative Y, we can’t get an A. So by MT, we will derive – A, like this:

1. ( A v B) & C Premise 1
2. A-> Y Premise 2
3. - ( Y & C ) Premise 3

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4. – Y v – C 3, DM

5. C 1, Simp

6. – Y 4, 5 DS

7. – A 2, 6 MT

Now, let’s not forget that the (AvB) in premise one is part of a conjunction that we can further simplify, like this:

1. ( A v B) & C Premise 1
2. A-> Y Premise 2
3. - ( Y & C ) Premise 3

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4. – Y v – C 3, DM

5. C 1, Simp

6. – Y 4, 5 DS

7. – A 2, 6 MT

8. A v B 1, Simp

Now, look at line 7 again. The negative A cancels out the A in the expression in line 8. So this means we are left with the letter B. Let’s write down this step:

1. ( A v B) & C Premise 1
2. A-> Y Premise 2
3. - ( Y & C ) Premise 3

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4. – Y v – C 3, DM

5. C 1, Simp

6. – Y 4, 5 DS

7. – A 2, 6 MT

8. A v B 1, Simp

9. B 7, 8 DS

There is nothing else we can do. So the letter B is the final conclusion.

Now your turn.

Derive a conclusion from the premises in each argument.

Don’t forget to write what method you used and from which lines you obtained the letters.

a)

1. ( A v B) & Q Premise 1
2. A-> Y Premise 2
3. - ( Y & Q ) Premise 3

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4.

5.

6.

7.

8.

9.

b)

\* Hint: start by taking apart the first premise. You should get in line 3 an expression that says: If this…., then that And if this…, then that…

1. W <-> T Premise 1. (If today is Wednesday, then tomorrow is Thursday

and if tomorrow is Thursday, then today is Wednesday.)

1. W Premise 2. Today is Wednesday.

\_\_\_\_\_\_\_

3.

4.

5.

c)

1. F <-> S Premise 1.
2. F Premise 2.

\_\_\_\_\_\_\_

3.

4.

5.

d)

1. H <-> A Premise 1.
2. H Premise 2.

\_\_\_\_\_\_\_

3.

4.

5.

e)

1. A <-> B Premise 1.
2. A Premise 2.
3. B -> C Premise 3.

\_\_\_\_\_\_\_

3.

4.

5.

6.

f)

1. ( M v W) & P Premise 1

2. M-> Y Premise 2

3.- ( Y & P ) Premise 3

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4.

5.

6.

7.

8.

9.

g)

1. - ( M & N)
2. M

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3.

4.